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Tangent Line

slope $\left(\frac{dy}{dx}\right)$

point

Find the equation for the tangent line at the given point

Q) $y = \frac{x^5 + 2x}{x^2}$ at $x = 1$

$\frac{dy}{dx} = \frac{x^2(5x^4+2) - (x^5+2x)(2x)}{x^4}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1^2(5(1)^4+2) - (1^5+2(1))(2\cdot 1)}{(1)^4}$

$\left.\frac{dy}{dx}\right|_{x=1} = 1$

$f(1) = 3$

$y = 3 + 1(x-1)$

R) $y = 5x^2 + 3$ at $x = 3$

$\frac{dy}{dx} = 10x$

$\left.\frac{dy}{dx}\right|_{x=3} = 10(3) = 30$

$f(3) = 5(3)^2 + 3$

$f(3) = 48$

$\frac{dy}{dx} = 30$

$x_1 = 3$

$y_1 = 48$

(3, 48)

$y = y_1 + \frac{dy}{dx}(x - x_1)$

$y = 48 + 30(x-3)$

S) Find an equation of the line perpendicular to the tangent to the curve $y = 4x^3 - 6x + 2$ at the point (2, 22).

normal line - opposite reciprocals

$\frac{dy}{dx} = 12x^2 - 6$

$\left.\frac{dy}{dx}\right|_{x=2} = 12(2)^2 - 6 = 42$

Tangent Line: $y = 22 + 42(x-2)$

Normal Line $y = 22 - \frac{1}{42}(x-2)$

T) Find the points on the curve $y = x^3 - 3x^2 - 9$ where the tangent is parallel to the x-axis (Horizontal Tangent)

$\frac{dy}{dx} = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0 \quad x = 2$

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U) Suppose u and v are differentiable functions at x = 2 and $u(2) = 3$, $u'(2) = 3$, $v(2) = 1$, $v'(2) = 2$

i) Find $\frac{d}{dx}(uv)$ $= u\left(\frac{dv}{dx}\right) + v\left(\frac{du}{dx}\right)$

$= u \cdot v' + v \cdot u'$

$= u(2) \cdot v'(2) + v(2) \cdot u'(2)$

$= 3 \cdot 2 + 1 \cdot 3$

$\left.\frac{d}{dx}(uv)\right|_{x=2} = 9$

ii) Find $\frac{d}{dx}\left(\frac{u}{v}\right)$

$\frac{v \cdot u' - u \cdot v'}{v^2} = \frac{v(2) \cdot u'(2) - u \cdot v'(2)}{[v(2)]^2} = \frac{1(3) - (3)(2)}{1^2}$

$\left.\frac{d}{dx}\left(\frac{u}{v}\right)\right|_{x=2} = -3$

iii) Find $\frac{d}{dx}(3u - 2v + 2uv)$ → Product Rule

$3\frac{du}{dx} - 2\frac{dv}{dx} + 2\left[u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}\right]$

V) Find the derivative of y = x with respect to x

W) Find the derivative of y = x with respect to t

X) Find the derivative of y = x with respect to P

Flowchart: Selecting a Procedure for Derivatives